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GAS SEPARATION IN THE RANQUE–HILSCH VORTEX TUBE

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Abstract—The gas separation taking place in the vortex tube is studied in detail. Both enrichment and depletion of a given component in any one of the two resultant streams may take place; the sign of this separation effect depends on certain parameters, notably the hot to cold flow ratio.

A comparison of the data shows how the pattern of the effect curve, i.e. the separation effect as a function of hot flow fraction, varies with constructional parameters. Among these the ratio of the diameters of the two orifices through which the gas escapes from the tube, is of paramount importance. Also their magnitude relative to the tube diameter has a distinct modifying effect. The separation ability as a function of the tube length has a maximum at quite short lengths, dependent, however, on the inlet jet diameter in such a way that an increase in this causes an increase in the optimal length.

The conclusion is reached that the centrifugation of the air, and only that, creates the gas separation detected in the outgoing streams. Its relation to the well-known temperature difference also produced between the two streams, is discussed.

A flow scheme involving the radial and axial flow components, which permit an interpretation and a correlation of the experimental data in a simple way, is put forward.

NOMENCLATURE

L,	length of vortex tube;
D,	diameter of vortex tube;
d_c,	diameter of cold orifice;
d_h,	diameter of hot orifice;
ε,	separation effect; here defined as the oxygen content of hot stream (through exit 4) minus that of cold stream (through exit 1), measured in per cent (v/v) absolute;
G,	total flow of gas through vortex tube;
θ,	hot flow fraction;
$\delta U =$	$G\theta(1-\theta)/2 \,.\, (\varepsilon/16)^2$ is separative power of vortex tube, $G\theta(1-\theta)/2 \,.\, [(N_h - N_c)/N(1-N)]^2$, where N_c and N_h are mole fractions of oxygen in the two streams, when the latter are approx. 0·2 (20% v/v oxygen) and ε is measured in per cent absolute.

INTRODUCTION

THE VORTEX tube, described by Ranque [1] and rediscovered by Hilsch [2] attracted attention primarily because of its peculiar ability to create temperature differences in a split gas stream.

The possibility that it may act as a separator of gas mixtures has also been investigated [3–10] and most authors conclude, though in some cases from rather inaccurate data, that separation does take place. There is, however, little agreement as to the reason why the two gas streams have different compositions, probably because a particular component (e.g. the heavier) has been found sometimes to be concentrated in the one stream and sometimes in the other.

In a preliminary work [11] it was shown how these diverging results may be explained in terms of variation of certain parameters, notably the length of the vortex tube and the diameters of its orifices, and it was indicated that the results pointed towards centrifugal effects (pressure diffusion effects) being at work. The present paper is an extension of this preliminary note. More data will be given in support of the theory and a full discussion will be included since only an abstract of the previous work is available.

In a following paper an attempt will be made to present a satisfactory flow mechanical picture of the secondary flows in the tube, which, as will be shown, are responsible for the complexity of the working mechanism.

The vortex tube in its simplest form is a piece of tubing closed towards one end by a plug, having a hole in the centre, and provided with a tangential inlet jet adjacent to the plug. A gas stream is led into the tube through the jet from a compressed air source whereby it whirls down the tube. By closing partly the far end of the tube, two gas streams are obtained, one through the centre hole in the plug, the "cold stream", and one through the partly closed end, the "hot stream". These designations are commonly used in discussions of the vortex tube and they will therefore be used here, although under special conditions the situation may be reversed (see below and [12]).

This model of vortex tube was found to be inconveniently inflexible for the present study. Instead the modification shown in Fig. 1 has been used. The essential changes are (1) that a plug with a hole in the centre, defines the hot end of the tube and (2) that valves in both exits make it possible to obtain any ratio of hot flow to cold without changing the constructional parameters. It has a further advantage in (3) that it is possible to obtain gas from the periphery or centre or both as desired.

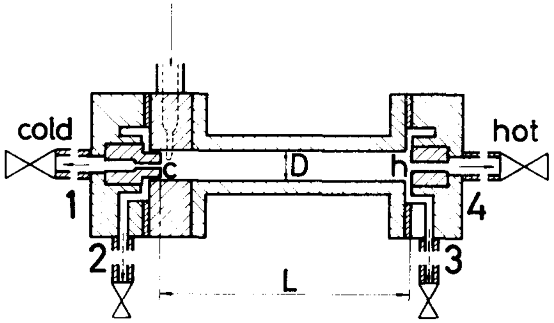

FIG. 1. Vortex tube design. The gas enters tangentially through the jet. It passes out through the cold and hot orifices at c and h (exit 1 and 4), or along the periphery and out through exit 2 and 3. All exits have valves and flow meters. The peripheral exits are, when not in use, closed by removing the spacers between main tube and end pieces. The different parts are held together by bolts.

In some experiments one of the orifices has been replaced by a coaxial tube of differing length. Also traditional vortex tubes according to Hilsch [2] and Stone and Love [3] have been tested.

The exit flows have been measured on flow meters.

The following list shows the parameters that have been varied while studying the effect. Some have been shown to be of secondary importance for the present purpose and further reference to those will be made only in so far as it may throw light on the mode of action of the tube; it is, however, the intention to present in later papers an account of the influence of such secondary parameters.

Constructional parameters:
Primary:

1. length of tube, L;
2. diameter of tube, D;
3. diameter inlet jet d_j;
4. diameter cold end orifice, d_c;
5. diameter of hot end orifice, d_h;
6. length of probe (if any) from cold end;
7. length of probe (if any) from hot end.

Secondary:

8. variation of D along the tube, including conical tubes;
9. jet design, including angle between jet and tube axis (normally 90°);
10. tube material (brass, plastic) including the condition of the tube wall (polished, greased), also the thickness of the wall;
11. hot end design;
12. distance from jet to cold end (normally very small).

3 different gas mixtures have been tested:

(a) oxygen and nitrogen = air;
(b) oxygen and carbon dioxide;
(c) oxygen and helium.

EXPERIMENTAL

The oxygen content of the two gas mixtures was, as in air, 20·9 per cent.

The gauge pressure of the compressed gas has, in all cases recorded here, been $3\frac{1}{2}$ atm.

The analytical procedure consisted in comparing the oxygen content of the different gas streams (in most cases the cold and hot streams only) on a Beckman Oxygen Analyzer, by alternately leading one or the other through the instrument.

The meter utilizes the change in magnetic susceptibility with changing oxygen concentration at constant temperature. Its sensitivity towards changes in oxygen concentration is independent of the absolute value of this concentration. The instrument available to us gives results that are accurate to within 1–2 10^{-3} per cent absolute of oxygen in the concentration range 20·9–21·0 per cent oxygen, that is to say the variations in the difference in oxygen content of two flows measured within 3–5 min are of this magnitude.

The possibility that the loss or gain of oxygen in any particular stream could be caused by chemical reaction with or diffusion through wall materials, has been eliminated by carrying out blank experiments and by performing recovery estimates (when possible).

The vortex tube provides remarkably reproducible separation effects, varying within one to three thousandths of a per cent absolute (SD $\frac{1}{2}$–1 $\times$ 10^{-3} per cent) when measurements are repeated within a few hours. Results obtained from experiments after an interval of $\frac{1}{2}$ year may show somewhat greater discrepancies, especially on the peak effects (up to 20 per cent relative). A shift in critical flow fraction (point of reversal of separation effect, see Figs. 2–4) up to about 10 per cent may also take place. Both changes show that additional parameters not under control, are at work. The very short (1 cm) vortex tubes are especially sensitive in this respect, one reason being that the flow is not completely axisymmetric even at the hot end, so that a small eccentricity in the position of the orifice may cause variation in the results.

RESULTS

A considerable amount of data has been collected, a fraction of which is reproduced in Figs. 2 to 4 (see also 7).

The following conclusions may be immediately drawn:

1. Positive as well as negative separation effects with well-defined peaks are created in many cases. When a vortex tube with a certain set of constructional parameters is capable of delivering effects of both signs, the positive effect always appear at low hot flow fraction and the negative at high.

2. The maximum and minimum become

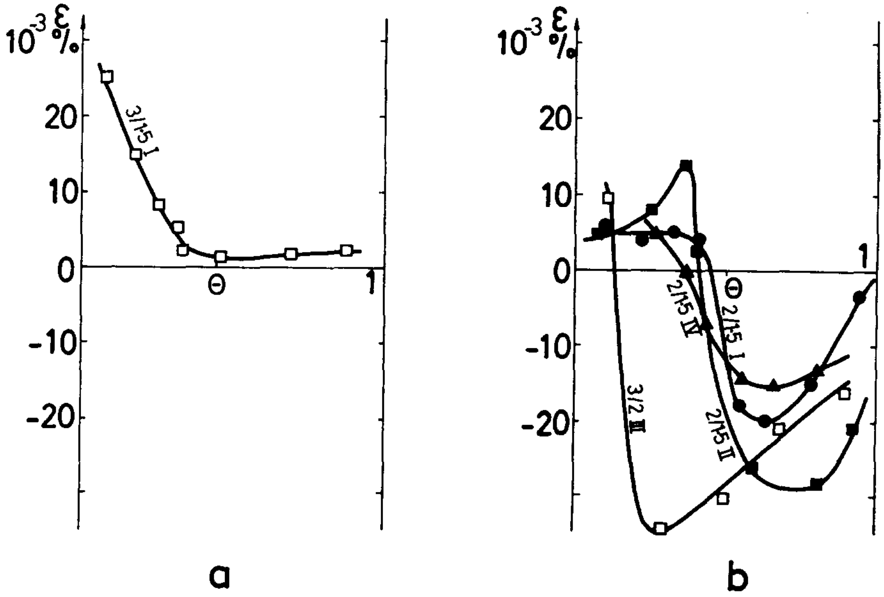

FIG. 2. Gas separation effect, ε, as a function of θ. Ratios indicate d_c (mm)/d_h (mm). $L = 1$ cm (○, ●); $L = 6$ cm (□, ■); $L = 13\frac{1}{2}$ cm (△, ▲). Compressed air (3·5 atm gauge pressure) was employed in all experiments shown. Material: Perspex tube and brass end pieces.

$d_c/d_h \geqslant 4/3$; (a) $D = 6$ mm; (b) $D = 10$ mm.

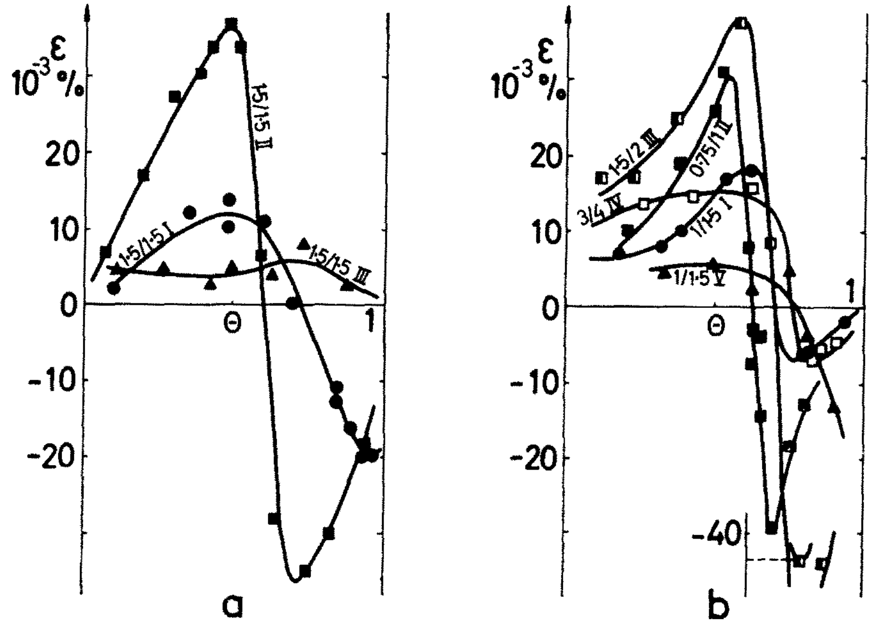

FIG. 3. (Cf. Fig. 2)

$2/3 < d_c/d_h < 4/3$; (a) $D = 6$ mm; (b) $D = 10$ mm.

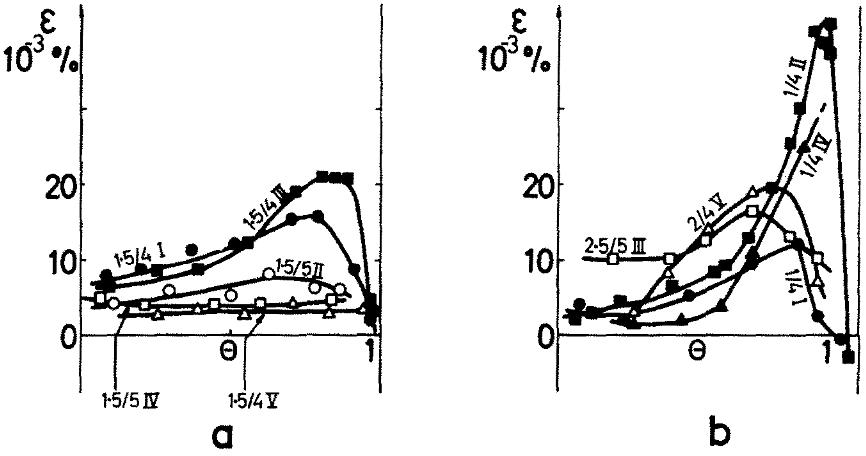

FIG. 4. (Cf. Fig. 2).

$d_c/d_h \leqslant 1/2$; (a) $D = 6$ mm; (b) $D = 10$ mm.

sharper the smaller the exit diameters [comp. Fig. 3(b) II–III–IV]. At larger orifices the typical pattern gradually is blurred and may finally disappear altogether [comp. Fig. 4(a) I–II or III–IV].

3. There is a dependence on the length of the vortex tube. The variation is quantitative as well as qualitative in that there is an optimal length around 6–10 cm (for tubes of diameter 6–10 mm) which provides the largest effects, and at the same time the longer tube does not exhibit as sharp maxima and minima on the curves as does

the shorter (comp. e.g. Fig. 2(b) I–II–IV or Fig. 4(a) I–III–V).

These impressions may be given a quantitative basis when the separative power concept of Cohen [13] is introduced: The separative power δU of the type of unit to which the vortex belongs is expressed as

$$\delta U = \theta(1-\theta)\,\tfrac{1}{2}\,G\left(\frac{\varepsilon}{16}\right)^2$$

where θ and $(1-\theta)$ are hot and cold flow fractions, G is number of moles per unit time introduced through the jet and ε is the separation effect in per cent absolute, as shown in Figs. 2–4, when the oxygen concentration is approximately 20 per cent. The results have been recalculated in terms of $\delta U/G$ and maximum values found for given sets of constructional parameters have been plotted versus tube length in Figs. 5(a) and (b).

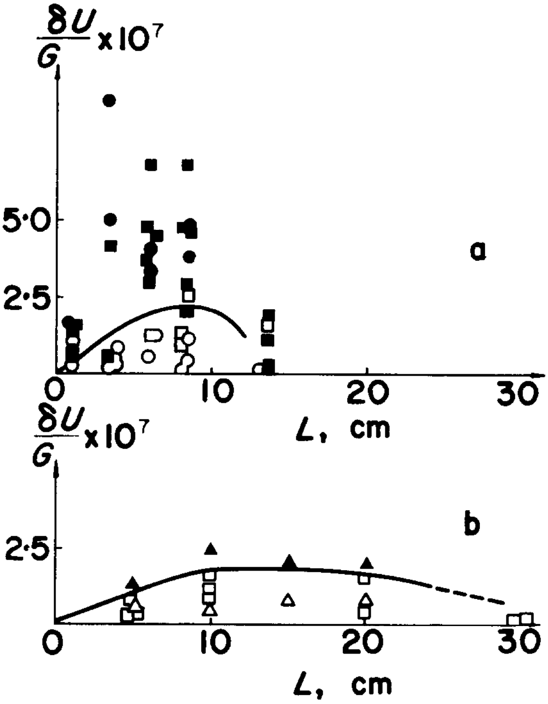

FIG. 5. $\delta U/G = \left[\frac{\theta(1-G)}{2}\left(\frac{\varepsilon}{16}\right)^2\right]_{\max}$ as a function of length of vortex tube, L. $D = 6$ mm (○, ●); 10 mm (□, ■); 15 mm (△, ▲); $(d_c + d_h)/D < 0·6$ (●, ■, ▲); $> 0·6$ (○, □, △)

A particular set of d_c and d_h is represented by one point only. (a) Jet diameter, $d_j = 1$ mm, $G \simeq 35$ l/min; (b) $d_j = 2$ mm, $G \simeq 130$ l/min.

The advantage of this kind of representation is that δU, as a measure of the separation ability of a tube, takes into account that it is easier to produce large effects when one of the resultant flows is small compared to the other than when they are of equal size.

More data are included than shown in Figs. 2–4, among them results obtained with copies of traditional types of vortex tubes, as used by Hilsch [2]. The results obtained with such tubes are very similar to those shown in Figs. 2–4 under comparable conditions, i.e. with d_h large and d_c small.

The data have been divided into two groups on the basis of a fixed value (0·6) of the parameter $(d_c + d_h)/D$, which takes into account the width of the orifices compared to the tube diameter. It will be seen that there is a separation of the two groups in the diagrams, and it is therefore concluded that $(d_c + d_h)/D$ is an important parameter for the separation ability. Furthermore it will appear from a study of Fig. 5 that the value of D, the diameter of the tube, has little or no influence on the separation ability. It should be added, however, that there is some indication that the complete effect curves [comp. Figs. 3(a) III and 4(b) V] are not so well developed for a long narrow tube as for an equally long wide tube.

A comparison of Figs. 5(a) and (b) shows that the jet diameter is an important parameter: For tubes with a larger jet and therefore a greater throughput of air the optimal length is considerably longer than for tubes with a narrow jet.

4(a). The most important parameter, as will be seen (Figs. 2–4) is the ratio of the diameters of the two orifices. A large cold to hot orifice ratio extends the region with negative effects, a small cold to hot orifice ratio favours the region with positive effect. When the orifices are of about the same size the curve is fairly symmetrical about the point, $\theta = \frac{1}{2}$ and zero effect.

Complete symmetry with respect to the two ends does not exist. This is especially evident [Fig. 2(a)] when the cold orifice becomes large while the hot is kept relatively small. The large negative effect which according to the above should result, is completely destroyed. On the other hand the tube is not symmetrical with respect to the jet and in the example mentioned the vanishing of the effect may be connected in some way to the fact that a large amount of air has to pass directly from the jet to the cold orifice.

4(b). The vortex tube is, as mentioned, provided with valves which permit any flow ratio to be obtained. An especially interesting situation arises with both valves completely open, i.e. with the back pressures of both orifices atmospheric. This characteristic flow ratio is determined by the diameter ratio of the orifices. Figure 6 shows that there is almost linearity between the hot flow fraction corresponding to zero effect and this "fully open" hot flow fraction. In actual fact, the latter is always found somewhere between the flow fractions of the maximum and minimum of the separation effect curve.

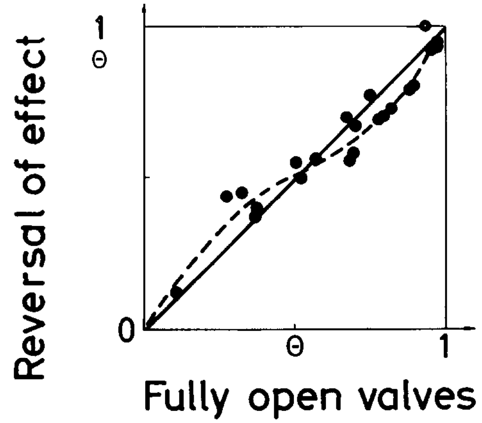

FIG. 6. Hot flow fraction corresponding to point of reversal of separation effect, as a function of hot flow fraction obtained with both valves fully open. Parameters varied are d_c and d_h, also D and L.

This result is very important, as the discussion will show, for the understanding of the mechanism responsible for the creation of the complex effect-curve pattern.

It has been possible to identify the region in which active separation occurs. This is illustrated in Fig. 7 which has been obtained with a tube furnished with 4 exits: A hot and cold orifice in the centres of the two plugs and an exit at each periphery of these plugs (see Fig. 1). The characteristic effect curve pattern is obviously a feature originating somewhere in the central part of the tube while the periphery is secondary in that respect.

It should be noted that the hot peripheral stream and the hot orifice stream are somewhat alike when the resulting effect is positive, i.e. when the heavier gas, oxygen, is concentrated in the hot stream, but are vastly different when it is negative.

The following Figs. 8(a) and (b) show the effect of replacing the orifices by tubular probes extending into the tube along the centre axis. Different lengths have been tested. It is seen that

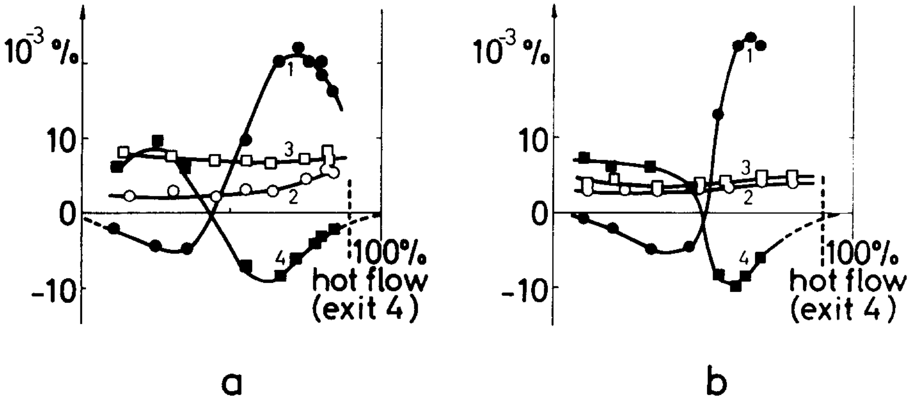

FIG. 7. Four-exit vortex tube (see Fig. 1). Oxygen concentration of exit flow (1, 2, 3 or 4) minus that of inlet flow as a function of hot flow fraction. Flow fractions through exits 2 and 3 were each approx. 5 per cent; hot flow fractions above 90 per cent not obtainable. $D = 10$ mm, $d_c = d_h = 2$ mm; (a) $L = 8\frac{1}{2}$ cm; (b) $L = 3\frac{1}{2}$ cm.

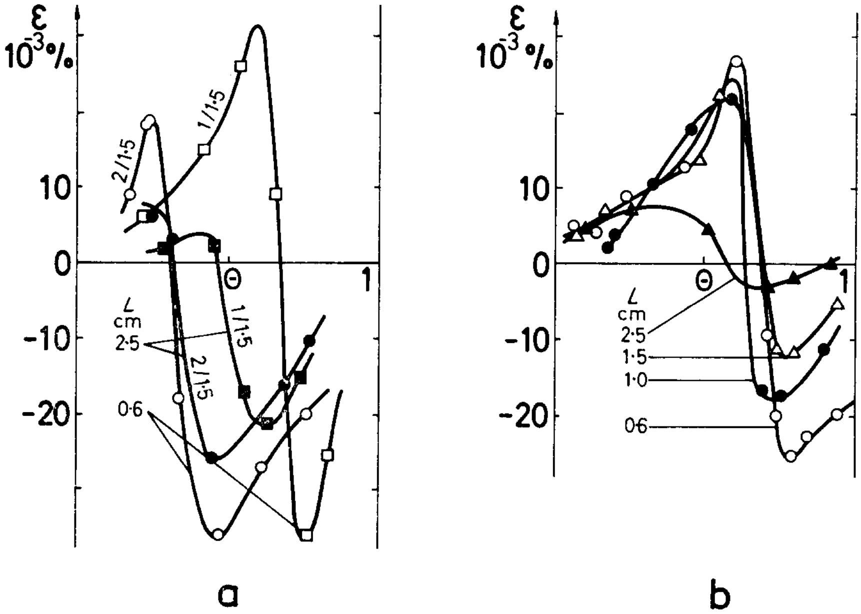

FIG. 8. Effect of substituting thinwalled, coaxial tube of varying length, l, for simple orifice. Ordinates and abscissas, cf. Figs. 2–4, $D = 10$ mm, $L = 8\frac{1}{2}$ cm.

(a) Tube protruding from hot end. Ratios indicate d_c (mm)/d_h (mm).

(b) Tube protruding from cold end. $d_c/d_h = 1{\cdot}5$ mm/2 mm.

the effect pattern is rather insensitive to this modification, from which follows that the effect is probably not generated right at the plug walls nor near the incoming jet.

It is characteristic that a hot end probe disturbs a positive effect more easily than a negative and vice versa. This places the origin of the positive effect near the hot end and that of the negative near the cold end.

It has also been tested whether the short ducts through the orifice plugs take part in the formation of the effect. A piece of wire was placed across the exit at the orifice. This is likely to cause turbulence in the duct and so disturb any flow pattern that may exist there. The resultant effect curves were almost identical to the one obtained normally.

It may be concluded that the region mainly responsible for creating the separation effect is a fairly narrow, coaxial cylinder within the tube.

Temperature effects

The correlation of isotope effects and temperature effects is a problem which deserves attention and it will be discussed in detail in a future publication. It should be stated at once that there is no direct connection between the two. Under ordinary conditions the hot end stays hot even though the separation effect changes sign. Furthermore, only small temperature effects arise in short tubes where the separation of gases is best while the best temperature effects are obtained in tubes so long that all traces of separation effects have disappeared.

It is, however, possible when the tube is sufficiently short to reverse also the temperature effect and for the very shortest tube lengths the two effect reversals take place at the same flow ratio, as shown on Fig. 9.

It is, in view of the above, not surprising to find that the temperature distribution in the tube is of minor importance to the separation effect created, as clearly shown on Fig. 10. Here two thick walled vortex tubes are compared, the one is made from brass the other from Perspex. Corresponding effect curves are almost identical but the brass tube gave no temperature effect at all, while the plastic tube gave up to about 20°C.

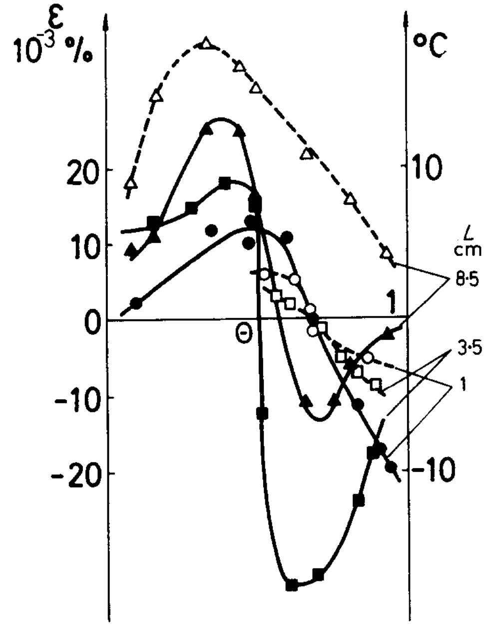

FIG. 9. Comparison of gas separation effect, ε (●, ■, ▲) with temperature effect (○, □, △). Right ordinates: Temperature of exit 4 minus temperature of exit 1, both measured after exit ducts. $D = 6$ mm, $d_c = 1{\cdot}5$ mm, $d_h = 1{\cdot}5$ mm.

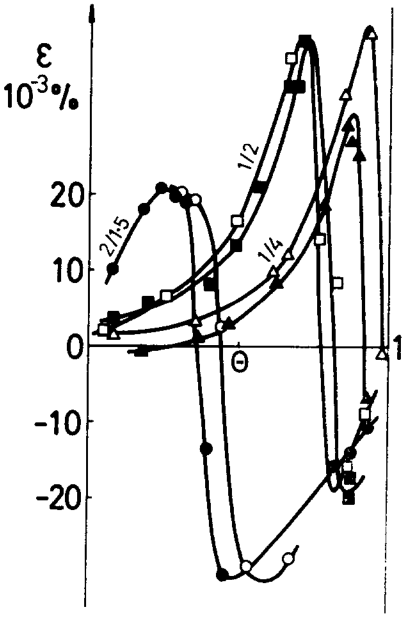

FIG. 10. Comparison of gas separation in Perspex tube (○, □, △) and thick walled brass tube (●, ■, ▲). $D = 10$ mm, $L = 8\frac{1}{2}$ cm. Ratios indicate d_c (mm)/d_h (mm).

DISCUSSION

The foregoing results show beyond any doubt that a separation effect is created and that it is remarkably reproducible, but, at least in the present case, fairly small. The driving force behind it has, as mentioned earlier, been a matter of controversy. The following list probably exhausts the relevant possibilities:

1. Effects due to centrifugal action = pressure diffusion.
2. Thermal diffusion effects.
3. Wall effects.
4. Effects due to deviation from normal equilibrium distribution of molecular velocities, especially near the jet.

The main problem in previous works has been the origin of the negative effects. The present results display a symmetry with respect to positive and negative effects which suggests that one and the same driving force is responsible for both; it would indeed be a surprising coincidence if two independent separation forces interacted systematically in such a way that the variation of constructional parameters such as jet design, tube diameter and length had little or no effect on the relative shape of the separation effect curves.

Possibility 3 and 4 may be immediately ruled out on the evidence of Figs. 7 and 8, which show that the main separation takes place towards the centre axis of the tube away from the peripheral wall and the jet.

Thermal diffusion effects will be closely related to the centrifugal field just as will the pressure diffusion effects. The temperature gradients that may be obtained, are, however, not greater than corresponding to an adiabatic expansion of the gas in the radial pressure field, and it is easily shown that an effect due to thermal diffusion, in that case, is an order of magnitude smaller than the corresponding centrifugation effect. Incidentally the data of Scheller and Brown [14] show that in a counter-current vortex tube there may be a higher static temperature at the centre than at the periphery; but apparently the gradient is likely to be smaller than the adiabatic gradient.

At any rate, the results in Fig. 10 clearly show that the separation effect is insensitive to radical changes in thermal boundary conditions, a finding which would be difficult to reconcile to the existence of any significant thermal diffusion effect.

The conclusion may thus be drawn that centrifugation effects, and only those contribute significantly to any net separation effect detected in the outgoing streams.

Figure 6 clearly linked the point of reversal of the separation effect to a flow dynamical parameter, so that it is reasonable to seek the explanation for the rather complex effect curve pattern in a study of the flows, especially the secondary, in the vortex tube. This is in fact possible and a combination of all available results has led to the conclusion that flow patterns as shown in Figs. 11(a) and (b) must exist in the vortex tube with net transfer of material leading to separation as indicated by arrows.

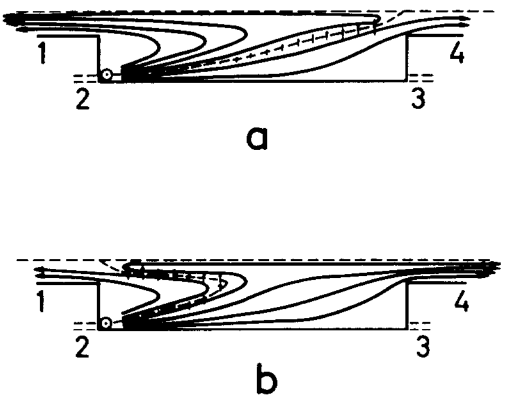

FIG. 11. Flow patterns; the result of a correlation of all data presented in the paper. (a) Hot flow heavier than cold. (b) Cold flow heavier than hot.

Net transfer of light component leading to separation indicated by arrows.

The following evidence supports this hypothesis:

1. The gas separation takes place along the line or rather surface that separates the two streams which afterwards emerge through the exit ducts. It consists in a radial diffusion of a net amount of the lighter component inward and the heavier component outward. The rate of transport is proportional to the square of the tangential velocity divided by the radius at any particular point as long as the concentration gradient is far from the equilibrium one; at equilibrium there is of course no net transport. The tangential velocity profile is a function of a number of vortex tube parameters including such that determine the degree of turbulence in the flow (see Kendall [15]).

In a following paper it will be shown that in the present case, in short tubes, the tangential velocity increases towards the centre, i.e. the radial Reynolds number (see [16]) is relatively large. The main region of separation is therefore towards the centre. Keyes [17] finds a similar velocity distribution and also conclude from theoretical and experimental evidence that the dominant separation takes place in the centre region.

Turbulence will reduce the equilibrium gradient obtainable (as described by Rosenzweig *et al.* [18]). In most of the tube except at the jet and near the peripheral wall the flow in itself is not unstable since the tangential velocity, v, is proportional to r^n (see Keyes [17]) where n is greater than minus one. A transition towards laminar flow is therefore to be expected as the gas moves inwards; especially in the region with forced vortex motion near the axis the stabilization may be considerable. This conclusion is also reached by Keyes [17] and it is in agreement with the experimental result obtained by Sibulkin [19].

The higher turbulence level in the outer part of the tube may well further reduce the limited gas separation taking place there; thus the contrast between separation ability of the centre region and of the peripheral region, as displayed by the results in Fig. 7, is fully accounted for by this model.

2. The existence of a centre region with reversed flow [Fig. 11(b)] has been observed by other authors [20, 21]. In the present case it was tested visually in crude experiments, (a) by observing the displacement of grease along the peripheral wall and (b) by placing small strips of plastic foil on a wire across the tube normal to the axis and observing their relative positions. It was found that the U-shaped flow pattern of Fig. 11(a) existed under conditions which favoured a positive separation effect, while flow ratios corresponding to conditions favouring

negative effects gave an S-shaped flow-pattern in agreement with Fig. 11(b). The negative effect itself was, however, destroyed by the wire (which was placed approx. one cm from the cold end), so that these findings can only serve as a guide to the present discussion.

3. It is easily seen that with the conclusion drawn in paragraph one that the centre region dominates the separation ability of the tube, the hot stream will be the heavier in case (a), i.e. positive effect, and the lighter in case (b), i.e. negative effect. Thus, the schemes in Fig. 11 may explain the appearance of effects of both signs; moreover, the change in sign is linked to a shift in flow pattern that may well be governed by changes in back pressures, outside the exit ducts. It is readily deduced from Figs. 6 and 2–4 that higher back pressure at the hot end than at the cold gives rise to positive effects while a pressure gradient in the opposite direction ensures a negative effect. When applied to the schemes in Fig. 11 these results are seen to indicate a plausible driving force behind the axial centre flows.

4. The evidence in Fig. 8 suggested a region of origin of the negative effect towards the cold end and that of the positive towards the hot end also in concord with Fig. 11.

5. This designation is furthermore supported by the conclusion drawn on the basis of the results in Figs. 2–4. Without a symmetrical build-up of the two types of flow pattern it would be difficult to explain the high degree of symmetry in effect curve shape with respect to positive and negative effects.

The observed smooth transition from positive to negative effect or vice versa, also find its explanation in the flow patterns of Fig. 11, where the change-over from the one type of flow to the other only affects a small part of the tube.

6. The limited separation taking place in the outer part of the tube (see paragraph 1 above) will, according to the scheme in Fig. 11, always provide a peripheral stream with a small amount of excess heavy component, in agreement with the results of Fig. 7.

7. The lack of direct correlation between temperature effects and separation effects is easily explained in terms of the foregoing results: The temperature effect is mainly created in the outer region of the tube (with its higher turbulence level) as also assumed in theories which try to explain this effect (see from experiments [16]). This means that the hot stream always stays hot according to the schemes in Fig. 11 even when the separation effect changes sign in agreement with what is found experimentally for long tubes (see Fig. 9).

The increasing correlation between the two types of effect when the tube is made shorter (Fig. 9) may perhaps be ascribed to a decrease in turbulence in the shortened tube, a decrease which makes the small temperature separation in the forced vortex near the centre more prominent (see [16]).

More evidence in support of the above hypothetic flow picture will be presented in the following publication, including a more detailed explanation of the complexity of the results as displayed in Figs. 2–4.

The centrifugation theory, as presented above also finds support when a comparison is made between results obtained with the three mixtures previously mentioned, O_2–N_2, O_2–CO_2 and O_2–He. Cohen's theory of gas centrifuges [13] makes such a comparison possible. It is based on the separative power concept, which has been used above (see Fig. 5). Cohen shows that the max. separative power of a centrifuge is

$$\delta U_{\max} = \rho D \left[\frac{\Delta M(\omega r)^2}{2\,RT}\right]^2 \frac{\pi L}{2}\ \mathrm{g/s}$$

where ρ is the density, D is the diffusion coefficient, ΔM is the molecular weight difference of the gas components, ωr is the tangential (peripheral) velocity, L is the length of the centrifuge. Table 1 shows that there is quite good agreement

Table 1. *Separative power* (*relative*)

	O_2/He	O_2/N_2	O_2/CO_2
$\delta U_{\max}$ (calc)	660	1	3·1
δU (exp)	800	1	1·7

$\delta U_{\max} = \rho D \left[\frac{\Delta M\,(\omega r)^2}{2\,RT}\right]^2 \frac{L\pi}{2}$ [see (13) and text]

$\delta U_{\exp} = \frac{\theta\,(1-\theta)}{2} G \left(\frac{\varepsilon}{16}\right)^2$ (cf. text below Fig. 5)

Obtained for Perspex vortex tube, $D = 10$ mm; $L = 8\frac{1}{2}$ cm; $d_c = 2$ mm; $d_h = 1{\cdot}6$ mm. Gauge pressure of compressed gas (before jet): 3·5 atm.

between experiment and theory. It is here taken into account that the peripheral velocity is limited to the velocity of sound in the mixture and that this is inversely proportional to the square root of the molecular weight and dependent on the ratio of the specific heats. The important parameter is, therefore, not ΔM as in an ordinary centrifuge but rather $\Delta M/M$.

It should be added that the typical effect curve pattern exists in all three cases and that the sign of the effects is in agreement with the centrifugation hypothesis.

The pressure ratio between periphery and centre is proportional to the density and to the square of the peripheral velocity, so that the pressure distribution is independent of the mixture (except for smaller changes in specific heat ratio); this probably explains why the flow patterns are similar and therefore the effects comparable, although the effects are much smaller than the optimal.

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Résumé—La séparation dans un gaz qui a lieu dans le tube à tourbillon est étudiée en détail. L'enrichissement et l'appauvrisement en un constituant donné dans n'importe lequel des deux jets résultants peuvent se produire à la fois; le signe de cet effet de séparation dépend de certains paramètres, notablement du rapport de l'écoulement chaud à l'écoulement froid.

Une comparaison des données montre comment la forme de la courbe représentative de l'effet, c'est-à-dire l'effet de séparation en fonction de la fraction de l'écoulement chaud, varie avec les paramètres de construction. Parmi ceux-ci, le rapport des diamètres des deux orifices à travers lesquels le gaz s'échappe du tube est d'une suprême importance. Leur gradeur relative au diamètre du tube a également un effet modificateur distinct. La capacité de séparation en fonction de la longueur du tube a un maximum pour des longueurs très courtes, dépendant, toutefois, du diamètre du jet d'entrée de telle façon que son augmentation provoque une augmentation de la longueur optimale.

On atteint la conclusion que la centrifugation de l'air, et seulement cela, crée la séparation dans le gaz détectée dans les jets de sortie. Sa relation avec la difference de température bien connue produite également entre les deux jets, est discutée.

Un schéma d'écoulement impliquant les composantes radiale et axiale de l'écoulement, qui permet de façon simple une interprétation et une corrélation des données expérimentales, est proposé.

Zusammenfassung—Die Gastrennung in einem Wirbelrohr wird im einzelnen untersucht. Dabei kann auch Anreicherung als auch Verarmung einer gegebenen Komponente in jedem der beiden resultierenden Ströme auftreten; das Vorzeichen dieses Trennungseffektes hängt von bestimmten Parametern ab, besonders vom Verhältnis der warmen zur kalten Strömung.

Ein Vergleich der Daten zeigt, wie sich der Verlauf der Einflusskurve, d.h. der Trennwirkung als Funktion des warmen Strömungsanteils mit den Konstruktionsparametern ändert. Dabei ist das Durchmesserverhältnis der beiden Öffnungen, durch die das Gas aus dem Rohr austritt, von grösster Wichtigkeit. Auch ihre relative Grösse zum Rohrdurchmesser hat einen deutlichen, modifizierbaren Einfluss. Die Trennfähigkeit als Funktion der Rohrlänge hat ein Maximum bei ganz kurzen Längen hängt aber noch vom Durchmesser der Einlaufdüse in der Weise ab, dass dessen Vergrösserung auch eine Vergrösserung der optimalen Länge bewirkt.

Man gelangt zu der Schlussfolgerung, dass die Zentrifugalbewegung der Luft, und nur diese, die Gastrennung hervorruft, die in den austretenden Strömen festzustellen ist. Der Zusammenhang mit der bekannten Temperaturdifferenz, die zwischen den beiden Strömen auftritt, wird diskutiert. Ein Strömungsschema, das die radialen und achsialen Stromkomponenten enthält und eine Interpretation und Korrelation der experimentellen Daten auf einfache Weise gestattet, wird eingeführt.

Аннотация—Подробно изучался отрыв газа в вихревой трубке. Может иметь место как увелечение, так и уменьшение данного компонента в любом из двух результирующих течений; знах этого воздействия зависит от определенных параметров, в особенности от соотношения между горячим и холодным потоками.

Сравнение данных показывает, как меняется вид кривой, описывающей функцию зависимости отрыва от доли горячего потока, в зависимости от конструктивных параметров. Бажнейшим из них является отношение диаметров двух отверстий, через которые происходит истечение газа. Сильно сказывается отношение этой величины к диаметру трубы. Зависимость отрыва от длины трубы имеет максимум при достаточно коротких длинах. Максимум зависит от входного диаметра струи таким образом, что его увелечение вызывает увелечение оптимальной длины.

Получен вывод, что именно центрифугирование воздуха создает отрыв газа, обнаруженный в выходящих потоках. Обсуждается зависимость отрыва от разности температур, возникающей межнду двумя потоками.

Предложена модель потока, имеющая радиальные и осевые компоненты, которая позволяет простым образом объяснить и увязать экспериментальные данные.